

Influence of Cooper pairing on the inelastic processes in a gas of Fermi atoms.

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Abstract

Correlation properties in ultracold Fermi gas with negative scattering length and its impact on the three-body recombination is analyzed. We find that Cooper pairing enhances the recombination rate in contrast to the decrease of this rate accompanying Bose-Einstein condensation in a Bose gas. This trend is characteristic for all interval of temperatures $T < T_c$.

The investigations of ultracold atomic gases give unique possibilities for studying quantum correlations and their role in the kinetics and dynamical properties of macroscopic systems. At early stage of these investigations it has been predicted that the rate of inelastic processes drastically decreases with the Bose condensation due to principal reconstruction of quantum correlations [1]. The local three-particle correlator K_3 , responsible for the recombination in a dilute gas, reduces with decreasing the temperature $T < T_C$ and the ratio $K_3(T)/K_3(T_C)$ becomes equal to $\sim 1/6$ as $T \rightarrow 0$. As the gas parameter na^3 increases the effect decreases and practically vanishes at the liquid densities (a is the scattering length).

For the first time, the effect has been observed in work [2]. The authors studied the decay kinetics of a ultracold ^{87}Rb gas in a trap under the BEC realization. The experimental results turned out in a qualitative and quantitative agreement with the theoretical predictions. In fact, these experimental results were ones of first evidences for the quantum correlation formation in the process of the BEC kinetics with a limited lifetime of the system.

The analogous phenomenon takes place in the 2D case at finite temperatures $T < T_C$ when the condensate is absent [3]. This result reflects a key role of the local correlations which are close to the case of a genuine condensate.

The condensate of Cooper pairs occurs in a two-component atomic Fermi gas with the attraction between particles at sufficiently low temperatures. It is interesting to reveal how the Cooper pairing and pair condensate affect the three-particle recombination rate. Analyzing this problem, we suppose that the attraction is a result of the Feshbah resonance for s-wave scattering particles. Herewith, the interaction is supposed to have nonzero value for the particles of different components only.

The results obtained in the present work lead to a priory not evident conclusion, namely, Cooper pairing results in enhancing the probability of three-particle recombination. To make the investigation more transparent, we consider the low density gas, assuming the following conditions

$$|a| k_F \ll 1 \quad \text{and} \quad r_0 k_F \ll 1 \quad (1)$$

where r_0 is the characteristic spatial size of the interparticle interaction.

In the case of negative scattering length $a < 0$ the weakly bounded dimers, typical for $a > 0$, are not created. The recombination in this case is accompanied by the formation of a molecule at a deep energy level. Completely three-particle character of the relaxation in the case of the negative scattering length $a < 0$, including unitary region, is demonstrated in the work [4]. The formation of the weakly bounded dimers in the case of the positive scattering length $a > 0$ changes the kinetics of the strongly bound molecule formation essentially [5]. The large released energy transfers to the kinetic energy of the molecule as a whole and to the third particle involved into the recombination process. Here the picture is analogous for Bose and Fermi particles and the transition rate is proportional to the three-particle correlator in the both cases. However, this correlator for Fermi particles demonstrates the increase with the Cooper pairing in contrast to its decrease in the BEC case.

2. Considering three-particle recombination in a low density Fermi gas with the creation of a molecule in a strongly bound state, we present the Hamiltonian of the system in the form

$$\hat{H} = \hat{H}_0 + \hat{H}' \quad (2)$$

where \hat{H}_0 is the Hamiltonian corresponding to elastic processes and \hat{H}' is the Hamiltonian corresponding to inelastic recombination processes, respectively,

$$\hat{H}' = \frac{1}{2} \sum_{\sigma \neq \sigma'} \int d^3 r_1 d^3 r_2 d^3 r_3 \left\{ \begin{aligned} & \hat{\psi}_m^+(\vec{r}_1, \vec{r}_2) \hat{\psi}_\sigma^+(\vec{r}_3) \times \\ & \times \hat{V} \hat{\psi}_\sigma(\vec{r}_1) \hat{\psi}_{\sigma'}(\vec{r}_2) \hat{\psi}_\sigma(\vec{r}_3) + h.c. \end{aligned} \right\} \quad (3)$$

In this expression $\hat{\psi}_m^+$ is the creation operator of a molecule

$$\hat{\psi}_m^+(\vec{r}_1, \vec{r}_2) = \sum_{\vec{q}} \exp\left(-i\frac{1}{2}\vec{q}(\vec{r}_1 + \vec{r}_2)\right) \varphi_m(\vec{r}_1 - \vec{r}_2) \hat{b}_{m\vec{q}} \quad (4)$$

where φ_m is the wave function of a molecule in its center-of-inertia frame. The Fermi operators of particle annihilation have the standard form

$$\hat{\psi}_\sigma(\vec{r}) = \sum_{\vec{k}} \hat{c}_{\vec{k}\sigma} \exp(i\vec{k}\vec{r}) \quad (5)$$

Conserving the pairwise structure of the interaction, we present \hat{V} in the form $\hat{V} = U(\vec{r}_1 - \vec{r}_2) + U(\vec{r}_3 - \vec{r}_2)$. From (4) it follows that $|\vec{r}_1 - \vec{r}_2|$ in (3) has a scale of the molecule size r_* . The characteristic value of $|\vec{r}_3 - \vec{r}_2|$ in (3) is close to r_* since in this case only the third particle should get the momentum compared in the magnitude with the momentum of the created molecule. Analyzing (3) in the frame which origin is at the center-of-inertia of the three-particle ensemble, we introduce the variables

$$\vec{\rho} = \vec{r}_1 - \vec{r}_2, \quad \vec{\rho}' = \vec{r}_3 - \vec{r}_2 \quad (6)$$

The bare vertex for a product of three Fermi operators on the right hand-side with taking (6), (1) and the transposition symmetry into account can be transformed as

$$\begin{aligned} & \exp[i\vec{k}_1\vec{r}_1 + i\vec{k}_2\vec{r}_2 + i\vec{k}_3\vec{r}_3] \\ & \rightarrow \frac{1}{2} \exp(i\vec{k}_1\vec{\rho} + i\vec{k}_3\vec{\rho}') \left[1 - \exp(-i(\vec{k}_1 - \vec{k}_3)(\vec{\rho} - \vec{\rho}'))\right] \approx \\ & \approx \frac{i}{2} \exp(i\vec{k}_1\vec{\rho} + i\vec{k}_3\vec{\rho}') (\vec{k}_1 - \vec{k}_3)(\vec{\rho} - \vec{\rho}') \end{aligned} \quad (7)$$

When the energy of a created molecule $E_m \gg \varepsilon_F$, the momentum of the third particle and the momentum of the molecule are practically the same in the absolute magnitude. Taking into account the relation

$$\frac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3 = \frac{1}{2}\vec{\rho} + \vec{\rho}'$$

for the vertex in (3), we have

$$\begin{aligned} & \Gamma(\vec{q}, \vec{k}_1, \vec{k}_2, \vec{k}_3) \\ & = \frac{i}{4} \int d^3\rho d^3\rho' \left\{ \exp\left[-i\vec{q}\left(\frac{1}{2}\vec{\rho} + \vec{\rho}'\right) + i(\vec{k}_1\vec{\rho} + \vec{k}_3\vec{\rho}')\right] \times \right. \\ & \quad \left. \times (\vec{k}_1 - \vec{k}_3)(\vec{\rho} - \vec{\rho}') \varphi_m(\vec{\rho}) U(\vec{\rho}') + \right. \\ & \quad \left. + (\vec{\rho} \rightleftharpoons \vec{\rho}') \right\} \end{aligned} \quad (8)$$

Considering the Fermi gas with the large magnitude of negative scattering length $|a| \gg r_0, r_*$ and conserving limitation (1), we face with a sharp increase of vertex Γ under conditions of the Feshbah resonance. In fact, for the singlet pairs a quasi resonance state is realized in the continuous spectrum for energy $\varepsilon \rightarrow 0$, whereas the real dimer bound state is absent. The solution of the Schroedinger equation for the two-particle problem demonstrates that, for $|\vec{\rho}| < r_0$, the wave function obtains an additional large factor $|a|/r_0$. The probability amplitude to find simultaneously three fermions in the volume $\sim r_*^3$ requires a factor $(|a|/r_0)^2$ as compared with the case of noninteracting gas (c.p. [6], [7], [8]). Note that $r_* \lesssim r_0$. Correspondingly, vertex (8) obtains effectively an additional factor $\sim (|a|/r_0)^2$.

The integral in (8) can be represented as a sum of two integrals. Each of them is factorized as a product of two integrals over $\vec{\rho}$ and $\vec{\rho}'$. Integrating in every case by parts and taking into account that $|\vec{k}_i| \ll |\vec{q}|$, we find.

$$\Gamma = \tilde{\Gamma}(\vec{q}) \frac{(\vec{k}_1 - \vec{k}_3) \vec{q}}{q^2} \quad (9)$$

$$\tilde{\Gamma}(\vec{q}) = \xi \frac{i}{2} \left(\frac{|a|}{r_0}\right)^2 q \left[\frac{d\varphi_m(q/2)}{dq} U(q) + \varphi_m(q/2) \frac{dU(q)}{dq} \right]$$

Here ξ is the numerical coefficient. In all cases we suppose the spherical symmetry of the functions $U(\vec{r})$, $\varphi_m(\vec{r})$. As a result, $\tilde{\Gamma}(\vec{q})$ depends on the absolute value of vector \vec{q} alone.

Considering the recombination transitions and, correspondingly, \hat{H}' as a small perturbation, the number of transitions per unit time takes the form ($\hbar = 1$)

$$W = 2\pi \sum_{i,f} \hat{\rho}_i |\hat{H}'_{fi}|^2 \delta(E_f - E_i) = \int dt \langle \hat{H}'(0) \hat{H}'(t) \rangle \quad (10)$$

$$\hat{H}'(t) = \exp(i\hat{H}_0 t) \hat{H}' \exp(-i\hat{H}_0 t)$$

Here the operator $\hat{\rho}_i$ is the equilibrium density matrix defined by the Hamiltonian \hat{H}_0 . Summing over index f in (10), we use the inequality $\varepsilon_F \ll E_m$. Then the block

$$\sum_{\vec{q}} |\tilde{\Gamma}(q)|^2 \frac{((\vec{k}_1 - \vec{k}_3) \vec{q})}{q^2} \frac{((\vec{k}'_1 - \vec{k}'_3) \vec{q})}{q^2} \int_{-\infty}^{\infty} dt \exp\left(-i\left(E_m - \frac{3}{4} \frac{q^2}{m}\right)t\right)$$

can be singled out in (10). Here the value $\frac{3}{4}(q^2/m)$ is the overall kinetic energy of the created molecule and fast atom with the momenta equal in magnitude and opposite in the direction. The direct calculation of this expression gives

$$\frac{82}{9\pi} q_* |\tilde{\Gamma}(q_*)|^2 \frac{((\vec{k}_1 - \vec{k}_3) (\vec{k}'_1 - \vec{k}'_3))}{q_*^2}$$

As a result, for the probability (10) we find

$$W = B \sum_{\substack{\vec{k}_i, \vec{k}'_i \\ i=1,2,3}} \frac{((\vec{k}_1 - \vec{k}_3) (\vec{k}'_1 - \vec{k}'_3))}{q_*^2} K^{(3)}(\vec{k}_i, \vec{k}'_i; \sigma, \sigma') \quad (11)$$

where $K^{(3)}$ is the three-particle correlator

$$K^{(3)}(\vec{k}_i, \vec{k}'_i) = \langle \hat{c}_{k'_{1\uparrow}}^+(0) \hat{c}_{k'_{2\downarrow}}^+(0) \hat{c}_{k'_{3\uparrow}}^+(0) \hat{c}_{k_{3\uparrow}}(0) \hat{c}_{k_{2\downarrow}}(0) \hat{c}_{k_{1\uparrow}}(0) \rangle \quad (12)$$

The value of the constant B is determined by the expression

$$B = \frac{4}{9\pi} m q_* |\tilde{\Gamma}(q_*)|^2 \quad (13)$$

The characteristic transition time $1/E_m$ is small compared with the characteristic correlation time in (11), and it determines effectively the correlator (12) for the coincident time moments.

3. We begin with the calculation of the probability W (11) at zero temperature $T = 0$. In the lack of interparticle interaction the correlator $K^{(3)}$ reads

$$K^{(3)}(\vec{k}_i, \vec{k}'_i) = n_{\vec{k}_{1\uparrow}} n_{\vec{k}_{2\downarrow}} n_{\vec{k}_{3\uparrow}} \left(\delta_{\vec{k}_1; \vec{k}'_1} \delta_{\vec{k}_3; \vec{k}'_3} - \delta_{\vec{k}_1; \vec{k}'_3} \delta_{\vec{k}_3; \vec{k}'_1} \right) \delta_{\vec{k}_2; \vec{k}'_2}$$

Substituting this expression into (11), we find

$$W_0 = B \frac{12}{5} \frac{k_F^2}{q_*^2} n^3, \quad (14)$$

where n is the particle density with the fixed spin projection. If the interaction between particles is taken into account, correlator (12) should be averaged over the changed ground state. We start from the case when the Cooper pairing is absent and then find the correlator $K^{(3)}$ in first order in the parameter $|a| k_F \ll 1$.

The interaction Hamiltonian for low energy particles can be written as

$$\hat{H}_{int} = - \sum_{\substack{\vec{p}_1, \vec{p}_2 \\ \vec{p}_1' \vec{p}_2'}} V(\vec{p}) \hat{c}_{\vec{k}_1' \uparrow}^+ \hat{c}_{\vec{k}_2' \downarrow}^+ \hat{c}_{\vec{k}_2 \downarrow} \hat{c}_{\vec{k}_1 \uparrow},$$

where $\vec{p} = \vec{p}_1' - \vec{p}_1$ and an additional condition for summing in this expression is $\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$. In first order in the interaction the wave function of the ground state can be represented as

$$\psi = \psi_0 + \sum_{S \neq 0} \frac{\langle S | \hat{H}_{int} | 0 \rangle}{E_0 - E_S} \psi_0^{(S)}$$

The direct calculation of the correlator $K^{(3)}$ with taking this reconstruction into account results in the appearance of four terms. If the factor $(\vec{k}_1 - \vec{k}_3)(\vec{k}_1' - \vec{k}_3')$ is involved, these terms give the identical summands in Eq. (11), and the correlator can be written as

$$K_1^{(3)}(\vec{k}_i, \vec{k}_i') = -V(\vec{k}) \left[\frac{n_{\vec{k}_1' \uparrow} n_{\vec{k}_2' \downarrow} (n_{\vec{k}_2 \downarrow} + n_{\vec{k}_1 \uparrow})}{\varepsilon_{\vec{k}_1'} + \varepsilon_{\vec{k}_2'} - \varepsilon_{\vec{k}_1} - \varepsilon_{\vec{k}_2}} + \frac{n_{\vec{k}_1' \uparrow} n_{\vec{k}_2' \downarrow}}{\varepsilon_{\vec{k}_1'} + \varepsilon_{\vec{k}_2'} - \varepsilon_{\vec{k}_1} - \varepsilon_{\vec{k}_2}} \right] n_{\vec{k}_3 \uparrow} \delta_{\vec{k}_3; \vec{k}_3'} \quad (15)$$

where $\vec{k} = \vec{k}_2' - \vec{k}_2$. In this expression the term with a product of four functions $n_{\vec{k}_i}$ in the numerator gives zero contribution to (11) due to the symmetry and can be omitted.

We use the usual method of substituting genuine potential $V(\vec{r})$ for an effective potential $\bar{V}(\vec{r})$ with the same magnitude of the scattering length a for small energies $\varepsilon \rightarrow 0$ and conserve a possibility to employ the perturbation theory. The potential $\bar{V}(\vec{r})$ can be taken as a simple spherical rectangular well with the depth $\bar{V}_0 \gg \varepsilon_F$ and the radius R_0 . Applicability of the perturbation theory implies the validity of the inequality $\varkappa R_0 \ll 1$ where $\varkappa = \sqrt{m\bar{V}_0}$, herewith, the bound state in the well is absent. It can easily be revealed that, for these requirements, the relation $R_0 = \eta |a|$ with $\eta \gg 1$ takes place as well as combined inequalities $k_F |a| \ll k_F R_0 \ll 1$. As a consequence, the Fourier component obeys the equality $\bar{V}(\vec{k} \rightarrow 0) \approx (4\pi/m)a \equiv g$ and the function $\bar{V}(\vec{k})$ begins drastically to decrease for $kR_0 \sim 1$ ($k \gg k_F$). The last point is important since with the substitution (15) into (11) and under the integration over momentum \vec{k} the divergent of the second term in the brackets in (15) is eliminated when the dependence $\bar{V}(\vec{k})$ is taken into account. The magnitude of the integral proves to be $\sim 1/R_0$ and the ratio g/R_0 becomes independent of the scattering length. In fact, the contribution of the corresponding term in the correlator $K^{(3)}$ determines some renormalization of the probability W_0 (14).

In the first term in (15) all the momenta obey inequality $k_i \lesssim k_F$ and the effective interaction can be approximated as $\bar{V}(\vec{k}) \simeq g$. Substituting $K_1^{(3)}$ into (11), we find the probability of the transition W_1 and, at the same time, the general expression for W in the normal phase in the linear approximation in interaction

$$W_n = W_0 + W_1 = W_0 \left(1 - \frac{6}{35\pi} (11 - 2 \ln 2) k_F |a| \right) \quad (16)$$

4. Consider now the contribution of the Cooper pairing to the formation of the three-particle correlator (12). For this purpose, averaging over the ground state, we should include the anomalous averages

$$K_S^{(3)} = - \langle \hat{c}_{\vec{k}_1' \uparrow}^+ \hat{c}_{-\vec{k}_1' \downarrow}^+ \rangle \langle \hat{c}_{\vec{k}_1 \uparrow} \hat{c}_{-\vec{k}_1 \downarrow} \rangle n_{\vec{k}_3} \delta_{\vec{k}_3; \vec{k}_3'} \delta_{\vec{k}_2'; -\vec{k}_1'} \delta_{\vec{k}_2; -\vec{k}_1} + \dots \quad (17)$$

In this expression there are additional three terms resulting from the commutations of the momenta and giving the identical contribution to (11).

Using the standard method of the u-v transformation for the calculation of the anomalous averages, see e.g. [9], we have at zero temperature

$$\langle \hat{c}_{\vec{k}_1 \uparrow} \hat{c}_{-\vec{k}_1 \downarrow} \rangle = \frac{\Delta}{E_{k_1}}$$

Here $E_{k_1} = \sqrt{|\Delta|^2 + \left(\frac{k_1^2}{2m} - \mu\right)^2}$ and Δ is the Cooper gap in the one-particle excitation spectrum. Within the framework of the method considered the integral in the expression obtained is well known

$$\int \frac{d^3 k_1}{(2\pi)^3} \frac{\Delta}{E_{k_1}} = \frac{2\Delta}{g}$$

One can see that the result holds for $T \neq 0$ if the gap Δ is the temperature-dependent quantity $\Delta(T)$. As a result, after substituting $K_S^{(3)}$ (17) into (11), we find directly

$$W_S(T) = W_0 \frac{9\pi^2}{4} \frac{1}{(k_F a)^2} \left(\frac{\Delta(T)}{\varepsilon_F} \right)^2 \quad (18)$$

From Eq. (18) it follows the main qualitative result: the Cooper pairing or condensate of Cooper pairs intensifies the three-particle recombination and thus the relaxation rate of the attractive Fermi gas. In this aspect the result differs in kind from the BEC case in a dilute atomic Bose gas in which, on the contrary, the rate of three-particle recombination decreases. Due to the behaviour of Δ as the function of the parameter $k_F a$ the three-particle recombination rate rises exponentially when $k_F |a|$ increases, at least up to $k_F |a| \lesssim 1$. The extrapolation of the results obtained from the region $k_F |a| \ll 1$ to the region $k_F |a| \lesssim 1$ demonstrates the real possibility of the experimental study of the effect.

The leading temperature correction for W_n is connected with W_0 and equal to $(\pi^2/3)(T/\varepsilon_F)^2$. Since $T_c \simeq \Delta(T=0)$, it is readily seen that, for the interval $T < T_c$, the temperature behavior of W is governed by the dependence $W_S(T)$ (18).

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